



 Coron Island

PHILIPPINE INTERNATIONAL MATH AND SCIENCE OLYMPICS NATIONAL ROUND 2022



PIMSO Math

GENERAL INSTRUCTIONS

- ① You have 90 minutes to finish the test.
- ② You may write your solutions on the TEST BOOKLET.
- ③ Write your answers on the ANSWER SHEET.
- ④ After the test, you must SUBMIT to the proctor both the TEST BOOKLET and the ANSWER SHEET.
- ⑤ This test covers FIVE (5) CATEGORIES namely: NUMBER THEORY, LOGICAL ANALYSIS, ALGEBRA, GEOMETRY, and COMBINATORICS.
- ⑥ There is a total of thirty (30) questions in this test. Each correctly answered question will be marked five (5) points. No point shall be deducted for incorrect answer.
- ⑦ You are NOT ALLOWED to use any calculating device during the test proper.
- ⑧ Any form of cheating is a ground for DISQUALIFICATION.

BEGIN HERE:

1. How many ways can you equally group 1,953,125 different objects such that nothing will be an excess?

2. What is the sum of the series below up to its 1000th term?

$$\frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{48} + \frac{1}{40} + \frac{1}{80} + \frac{1}{60} + \dots$$

3. Consider the expression $\frac{2^{k_1} - 2^{k_2}}{2^{k_3} - 2^{k_4}}$ where k_1, k_2, k_3, k_4 are all non-negative integers. What is the smallest value of n where n represents the positive integers that cannot be equal to the expression?
4. The passcode to Henry's computer is described in a hint: In a collection of 9809 distinct objects, there are N ways to arrange their order. The number of zeros at the end of N is the passcode. What is the passcode?
5. Jason partially forgot his 6-digit passcode. He correctly remembers that four of the digits are 1,2,3, and 4, in that specific order, but forgets their respective places in the passcode. He also remembers that the missing 2 digits are the same. Given this information, how many possible combinations of Jason's passcode are there?

10. If the 9th day of January 202X is three days earlier than Friday, what will be the 365th day of the year 202X?

11. Families living in New Clark City love biking. There are a total of 2,022 families in the city each with two or four bicycles. Given that there are as many families owning either four bicycles as many as families owning two bicycles, how many bicycles are there in New Clark City?

12. What is the value of CVCC?

$$\begin{array}{r}
 V \quad V \quad D \\
 C \quad D \quad V \\
 D \quad C \quad C \\
 \hline
 C \quad V \quad V \quad C
 \end{array}
 +$$

13. What is the sum of the roots of $ax^2 - 2022x - 2023 = 0$ if $a = 2$?

14. Let e be Euler's constant, an irrational number equal to 2.71828 ... Let e^2 and e be the roots of $x^2 - bx + c = 0$. Find the real number $b + c$.

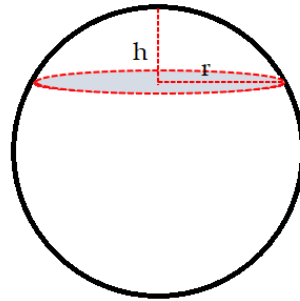
15. Let r_1 and r_2 be the roots of the equation $x^2 - 2022x + 2022 = 0$. What is the sum of the squares of the reciprocals of r_1 and r_2 ?

16. Let $f(x) = \frac{e^x - e^{-x}}{2}$ and $g(x) = \frac{e^x + e^{-x}}{2}$. Find $\frac{2e}{e^2 - 1} [f(g(0))g(f(0))]$.

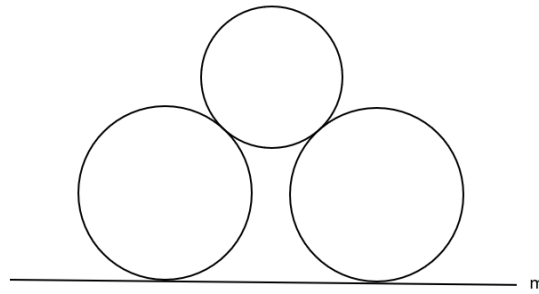
17. Let R and F ($R, F \neq 0$) be the roots of the equation $x^2 - 3x - m = 0$. If RF and $R + F$ are the roots of the equation $(c + 1)x^2 - sx + m = 0$ and $c + 1 + s + m = 0$, then find the value of $c + s - m$.

18. If $S = \frac{10}{100} + \frac{200}{100^2} + \frac{3,000}{100^3} + \frac{40,000}{100^4} + \dots + \frac{9,000,000,000}{100^9} + \frac{100,000,000,000}{100^{10}}$, what is $S + \frac{1}{10^9}$?

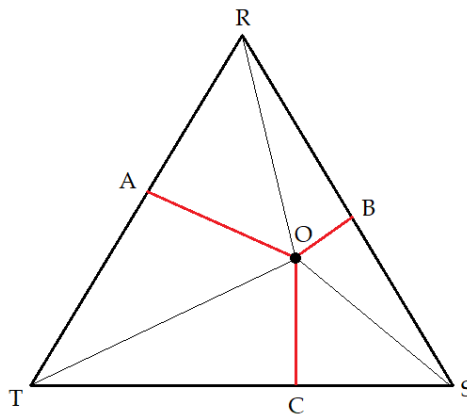
19. Rose fills a spherical fishbowl with water up to the level indicated in the figure below. The radius of the entire sphere is 10 inches. If the height of the unfilled portion is $h=2$ inches while radius $r=6$ inches, what is the volume of the water in the fishbowl? (Leave π as it is or express your answer using π .)



20. In the given problem in #19, the unfilled portion of the bowl is called the spherical cap. Find its surface area. (Leave π as it is or express your answer using π .)
21. Given the figure below, the distance between the center of the 2 identical bigger circles is given by x while their radii are given by R . If the radius of the smallest circle at the top is given by r , what is the shortest distance from the center of the topmost circle to line m ?



22. Consider the figure below (not drawn to scale). O is a point inside equilateral triangle RST such that $OA=27$, $OC=25$, and $OB=23$ are all perpendicular to the sides of RST . Compute for the area of RST . (Leave your answer in surd form.)



23. Circles A and B are tangent to each other at point X. Line segment CD is a direct common tangent to circles A and B at points C and D respectively. Find $\angle CDX$ if $\angle XCD=40^\circ$.

24. A certain semi-circular tunnel covers 2 cemented lanes. Beyond the lanes are the gutters whose widths of 1 ft. each extend up to each of the edges of the tunnel. The maximum height

of vehicles that can pass through the tunnel while travelling along the edge of the lane is $4\sqrt{2}$ ft. What is the total width of the tunnel?

25. There are 10 CISMO students in a classroom. It was agreed that all of the students' cellphones will be put to silent mode and placed on the proctor's table before the start of the examination. When the competition is over, everyone picks up a cellphone at random. What is the probability that nobody gets his/her own cellphone?
26. What is the coefficient of the third term of the binomial expansion $(a + 5b)^3$?
27. What is the coefficient of the seventh term of the binomial expansion $(a + 2b)^{10}$?
28. What is the sum of the coefficients of the first term and last term of the binomial expansion $(a + 3b)^8$?
29. What is the value of $a+b$ in the binomial expansion $(ax + by)^{10}$ if the coefficient of the 3rd term of the binomial expansion of $(ax + by)^9$ is 36 and if the coefficient of the 4th term of the binomial expansion of $(ax + by)^9$ is equal to 84?

30. If $f(0) = 1$ and $f(n + 1) = (n + 1)f(n)$, find $f(7)$.

-END OF TEST-

Thank you for participating in the PIMSO National Round!