



 Coron Island

PHILIPPINE INTERNATIONAL MATH AND SCIENCE OLYMPICS NATIONAL ROUND 2022



PIMSO Math

GENERAL INSTRUCTIONS

- ① You have 90 minutes to finish the test.
- ② You may write your solutions on the TEST BOOKLET.
- ③ Write your answers on the ANSWER SHEET.
- ④ After the test, you must SUBMIT to the proctor both the TEST BOOKLET and the ANSWER SHEET.
- ⑤ This test covers FIVE (5) CATEGORIES namely: NUMBER THEORY, LOGICAL ANALYSIS, ALGEBRA, GEOMETRY, and COMBINATORICS.
- ⑥ There is a total of thirty (30) questions in this test. Each correctly answered question will be marked five (5) points. No point shall be deducted for incorrect answer.
- ⑦ You are NOT ALLOWED to use any calculating device during the test proper.
- ⑧ Any form of cheating is a ground for DISQUALIFICATION.

BEGIN HERE:

1. If you're going to cross out all integers from 1 to 10,000 that are divisible by 3, 5, or 7 and encircle the rest, how many integers will be encircled?

2. What is the sum of this series up to its 100th term?

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{1}{12} + \frac{1}{10} + \frac{1}{20} + \frac{1}{15} + \dots$$

3. Consider the base-20 system using the first 20 letters of the Alphabet below.

| | | | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| A | B | C | D | E | F | G | H | I | J |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| K | L | M | N | O | P | Q | R | S | T |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

How will you write 9999_{10} using this base-20 system?

4. Using the base-20 system in #3, what is **BFLL** minus **FBB** in the binary system?

9. Pointing out to a man Robert Fordan said, "He is the son of a woman who is the mother of the husband of my mother." The man is Robert Fordan's _____.

10. In the 2022 PIMSO Math National examination, a student attempted 23 questions and got 45 points. If for every correct answer 3 points were awarded and for every wrong answer 1 point was deducted, then how many answers were wrong?

11. At the end of a meeting, seven students shake hands with each other once. How many handshakes will be there altogether?

12. Robert is at 10th position from the top. In the class of 50 students, there are 26 students between Migs and Robert. How many students are there in between Migs and the last student in the class?

13. If r and s are roots of the quadratic equation $x^2 - x - 1 = 0$, find $r + s$.

14. Let $f(x) = 2x + 1$ and $h(x) = x^3$. Find $f(h(0))h(f(0))$.

15. In the half-open interval $[0, 100)$, how many real numbers are integers?

16. Let R and F ($R, F \neq 0$) be the roots of the equation $x^2 - 3x + 2 = 0$.
If RF and $R - F$ are the roots of the equation $(p + 1)x^2 - mx + s = 0$, find $p + m - s$.

17. Simplify $3 + 6 + 9 + 12 + \dots + 2019 + 2022$.

18. What is the sum of the roots of $ax^2 - 2022x - 2023 = 0$ if $a = 2$?

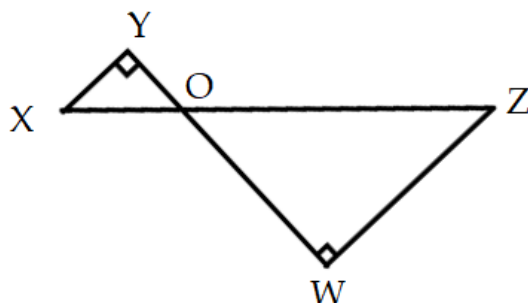
19. _____ is the point of intersection of the three altitudes of a triangle.

20. A rectangle lies on 2 intersecting identical circles such that the points of intersection of the 2 circles are also intersected by the rectangle. The radii of the circles are 61 mm each while the width of the rectangle is 120 mm. What is the distance between the centers of the 2 circles?

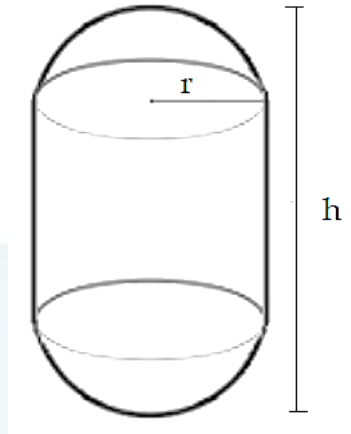
21. The lengths of the sides of triangle ABC are 10, 15, x with the possibility of being isosceles. What are the possible values of x if x may also be the shortest side?

22. Three circles are tangent to the same side of line M in such a way that all of the three are also tangent to one another. Suppose that the radii of the bigger circles are 25 and 16 units, what is the radius of the smallest circle?

23. Given the figure below (not to scale), find the length of WZ if $XY=5$, $YW=220$, and $XZ=221$.



24. Find the volume of the capsule below when it is $\frac{3}{4}$ full if $r=2$ inches and $h=10$ inches. (Leave π as it is or express your answer using π .)



25. There are 5 CISMO students in a classroom. It was agreed that all of the students' cellphones will be put to silent mode and placed on the teacher's table before the start of the class. When the class is over, everyone picks up a cellphone at random. What are the chances that nobody gets his/her own cellphone?

26. How many integers from 1 to 2,022 are not divisible by 2, 3 and 5?

27. What is the coefficient of the term of $f(x)$ containing x^2 if $f(x) = (2x + 2)^5$?

28. What is the coefficient of the fourth term of the binomial expansion $(2x + 2y)^{10}$?

29. What is the coefficient of the third term of the binomial expansion $(a + 5b)^3$?

30. If $f(0) = 1$ and $f(n + 1) = (n + 1)f(n)$, find $f(7)$.

-END OF TEST-

Thank you for participating in the PIMSO National Round!