



 Coron Island

PHILIPPINE INTERNATIONAL MATH AND SCIENCE OLYMPICS NATIONAL ROUND 2022



PIMSO Math

GENERAL INSTRUCTIONS

- ① You have 90 minutes to finish the test.
- ② You may write your solutions on the TEST BOOKLET.
- ③ Write your answers on the ANSWER SHEET.
- ④ After the test, you must SUBMIT to the proctor both the TEST BOOKLET and the ANSWER SHEET.
- ⑤ This test covers FIVE (5) CATEGORIES namely: NUMBER THEORY, LOGICAL ANALYSIS, ALGEBRA, GEOMETRY, and COMBINATORICS.
- ⑥ There is a total of thirty (30) questions in this test. Each correctly answered question will be marked five (5) points. No point shall be deducted for incorrect answer.
- ⑦ You are NOT ALLOWED to use any calculating device during the test proper.
- ⑧ Any form of cheating is a ground for DISQUALIFICATION.

BEGIN HERE:

1. What will be the remainder when $1! + 2! + 3! + 4! + 5! + \dots + 1000!$ is divided by 10?

2. What is the sum of the series below up to its 1000th term?

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{1}{12} + \frac{1}{10} + \frac{1}{20} + \frac{1}{15} + \dots$$

3. How many positive integer divisors does 25000 have?

4. Drake partially forgot his 6-character passcode. He correctly remembers that four of the characters are A, B, C, and D, in that specific chronological order, but forgets their respective places in the passcode. How many ways can Henry enter 1, 2, 3, 4?

5. If you're going to write down all integers from 100 to 1000 that are NOT divisible by 2, 5, or 7, how many will there be on the list?

6. What is the highest possible value of integer k that will make $100000!$ divisible by 661^k ?

7. [lan] The radius of circle A is $\frac{1}{3}$ the radius of circle B. Circle A rolls around circle B one trip back to its starting point. How many times will circle A rotate in total?

8. Complete the series: 13, 25, 1, 37, -11, 49, ____.

9. If the 9th day of January 202X is three days earlier than Friday, what will be the 365th day of the year 202X?

10. Families living in New Clark City love biking. There are a total of 2,022 families in the city each with two or four bicycles. Given that there are as many families owning either four bicycles as many as families owning two bicycles, how many bicycles are there in New Clark City?

11. What is the value of CVCC?

$$\begin{array}{r}
 V \ V \ D \\
 C \ D \ V \\
 D \ C \ C \\
 \hline
 C \ V \ V \ C
 \end{array}
 +$$

12. Vladimir's wallet contains 20, 50, and 100-peso bills. If the total value of the bills is Php1,650 and there are 30 bills, how many combinations of bills are possible?

13. Let R and F ($R, F \neq 0$) be the roots of the equation $x^2 - 3x + 2 = 0$.
 If RF and $R - F$ are the roots of the equation $(c + 1)x^2 - sx + m = 0$, find $c + 1 + s + m$.

14. Let R and F ($R, F \neq 0$) be the roots of the equation $x^2 - 3x - m = 0$.
 If RF and $R + F$ are the roots of the equation $(c + 1)x^2 - sx + m = 0$ and $c + 1 + s + m = 0$, then find the value of $c + s - m$.

15. Simplify $S = \frac{1}{10} + \frac{2}{10^2} + \frac{3}{10^3} + \frac{4}{10^4} + \frac{5}{10^5} + \frac{6}{10^6} + \frac{7}{10^7} + \frac{8}{10^8} + \frac{9}{10^9}$

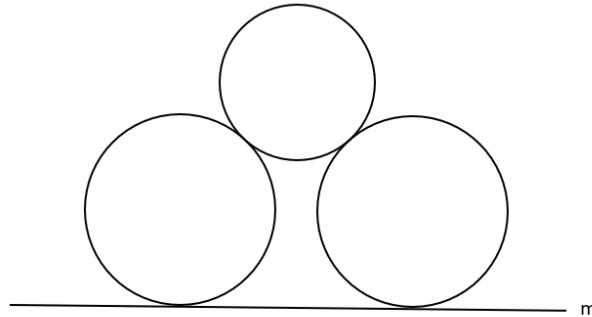
16. If $S = \frac{10}{100} + \frac{200}{100^2} + \frac{3,000}{100^3} + \frac{40,000}{100^4} + \dots + \frac{9,000,000,000}{100^9} + \frac{100,000,000,000}{100^{10}}$, what is $S + \frac{1}{10^9}$

17. Simplify $\frac{22}{70} \left(\frac{7}{4} + \frac{7}{12} + \frac{7}{24} + \frac{7}{40} + \frac{7}{60} + \frac{7}{84} + \dots + \frac{7}{180} + \frac{7}{220} \right)$

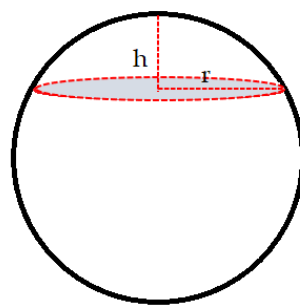
18. Let $f(x) = 2x + 1$ and $h(x) = x^3$. Find $f(h(0))h(f(0))$.

19. A rectangle lies on 2 intersecting identical circles such that the points of intersection of the 2 circles are also intersected by the rectangle. The radii of the circles are 85 cm each while the width of the rectangle is 168 cm. What is the distance between the centers of the 2 circles?

20. Given the figure below, if the distance between the center of the 2 identical bigger circles is 16 cm, while their radii are 6 cm each. If the radius of the smallest circle at the top is 3 cm, what is the shortest distance from the center of the topmost circle to line m ? (Leave your answer in surd form.)



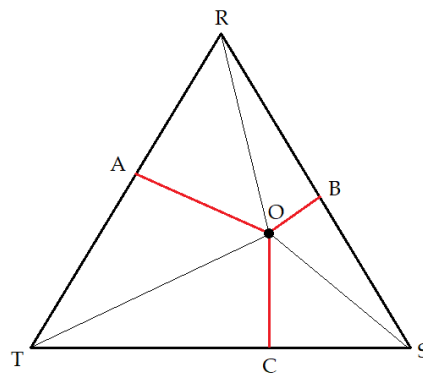
21. Inojin fills a spherical bowl with water up to the level indicated in the figure below. The radius of the entire sphere is 10 inches. If the height of the unfilled portion is $h=2$ inches while radius $r=6$ inches, how much water could Inojin still pour into the bowl? (Leave π as it is or express your answer using π .)



22. In the given problem in #21, the unfilled portion of the bowl is called the spherical cap. Find its surface area. (Leave π as it is or express your answer using π .)

23. GEOM is a square that encloses 2 circular arcs A and B such that A is centered at E and intersects G and O while B is centered at M and intersects G and O. Diagonal EM passes through the arcs at points C and D. Given that $CD = 30 - 15\sqrt{2}$ units, what is the perimeter of the square?

24. Consider the figure below. O is a point inside equilateral triangle RST such that $OA=17$, $OB=15$, and $OC=13$ are all perpendicular to the sides of RST. Compute for the area of RST. (Leave your answer in surd form.)



25. If $f(0) = 1$ and $f(n + 1) = (n + 1)f(n)$, find $f(7)$.

26. How many integers from 1 to 2,022 are not divisible by 2, 3 and 5?

27. There are 10 CISMO students in a classroom. It was agreed that all of the students' cellphones will be put to silent mode and placed on the proctor's table before the start of the examination. When the competition is over, everyone picks up a cellphone at random. What is the probability that nobody gets his/her own cellphone?

28. What is the coefficient of the third term of the binomial expansion $(a + 5b)^3$?

29. What is the coefficient of the seventh term of the binomial expansion $(a + 2b)^{10}$?

30. What is the sum of all the coefficients of the terms of the multinomial expansion of $(a + b + c)^{10}$?

-END OF TEST-

Thank you for participating in the PIMSO National Round!